Basic Concepts in Metric Spaces

1. Open Ball
Let (X, d) be a metric space and $a \in X$. For $\varepsilon > 0$, the open ball with center a and radius ε is defined as
$B(a, \varepsilon) = \{x \in X : d(x, a) < \varepsilon\}$.
Example: In R with usual metric $d(x, y) = |x - y|$, the open ball $B(2, 1) = (1, 3)$.

2. Closed Ball
The closed ball with center a and radius ε is defined as
B■$(a, \varepsilon) = \{x \in X : d(x, a) \leq \varepsilon\}$.
Example: In R, B■$(2, 1) = [1, 3]$.

3. Neighborhood of a Point
A set $N \subseteq X$ is called a neighborhood of a point $a \in X$ if there exists $\varepsilon > 0$ such that $B(a, \varepsilon) \subseteq N$.
Example: The interval (1, 4) is a neighborhood of 2 in R.

4. Open Set
A subset G of X is called an open set if every point of G is an interior point of G.
Example: The interval (1, 3) is an open set in R.

5. Interior Point
A point $a \in A \subseteq X$ is called an interior point of A if there exists $\varepsilon > 0$ such that $B(a, \varepsilon) \subseteq A$.
Example: 2 is an interior point of the set (1, 3).

6. Interior of a Set
The interior of a set A, denoted by A°, is the set of all interior points of A.
Example: If $A = [1, 3]$, then $A^\circ = (1, 3)$.

7. Closed Set
A set $F \subseteq X$ is called closed if its complement $X - F$ is open.
Example: The interval [1, 3] is a closed set in R.

8. Closed Point of a Set
A point $a \in A$ is called a closed point of A if its complement is an open set containing all other points except a.
Equivalently, singleton sets {a} are closed in metric spaces.
Example: Any single point {2} in R is a closed set.