For Stamp

(4) General equation of wave

Consider a transverse wave in a String that starts at A and Propagates from left to right along +ve x- direction as shown in fig-6. The Particle on the right will start vibrating after a Certain time as compared to the Particle on the left. Since every Particle of the medium Performs Simple harmonic motion, The equation of motion of any Particle, say A, is given by

$$y = a \sin \omega t$$

where a is the amplitude of the vibrating Particle, y is the displacement and angular velocity after time t.

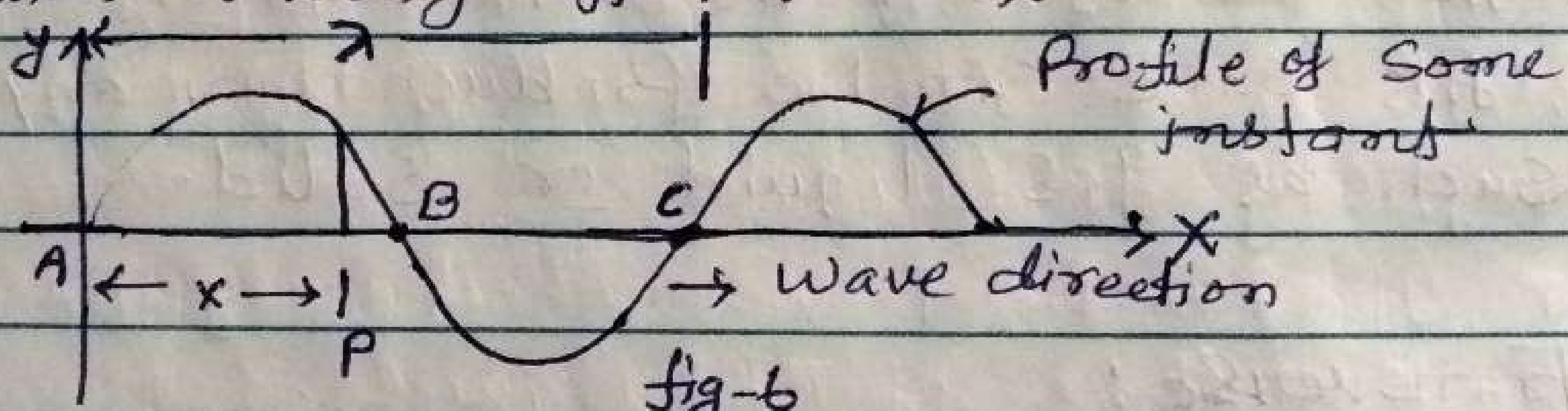

fig-6

If the frequency of vibration is n, Then $\omega = 2\pi n$

$$\therefore \; y = a \sin 2\pi n t$$

when Particle A Passes Through its mean Position, Then Particles B, C, etc. also Pass Through Their mean Position in the Same direction. So The Particle A, B, C... etc are in the Same Phase. The distance between two Consecutive Particles in the Same Phase is Called wavelength and on moving from A to B, The Phase changes by 2π. Therefore, on moving from Point A to Point P at a distance x from A, the Phase changes and is given by ϕ-

$$\phi = \frac{2\pi}{\lambda} x$$

Hence The displacement of P is given by :

$$y = a \sin(\omega t - \phi)$$

$$= a \sin\left(2\pi n t - \frac{2\pi}{\lambda} x\right) = a \sin\left(2\pi \frac{V}{\lambda} t - \frac{2\pi}{\lambda} x\right)$$

or $$y = a \sin \frac{2\pi}{\lambda}(Vt - x) \quad \text{——} \quad (1)$$

The equation is quite general and gives the displacement of particles whose distance x from a fixed Point A is known at any time. So ~~that~~ This is the equation for an increasingly Simple harmonic wave. The number of wave-lengths in a unit distance is called wave-number. It is denoted by $\bar{\nu}$.

$$\therefore \bar{\nu} = \frac{1}{\lambda} \quad \text{——} \quad (2)$$

The quantity $2\pi/\lambda = K$ is called Propagation constant.

<u>Other form</u>: eqn (1) can also be written as

$$y = a \sin\left(\frac{2\pi V}{\lambda} t - \frac{2\pi}{\lambda} x\right) = a \sin(\omega t - Kx) \quad \text{——} \quad (3)$$

$$\left[\because \frac{2\pi V}{\lambda} = 2\pi n = \omega\right]$$

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In exponential form

$$y = ae^{i(\omega t - Kt)} \longrightarrow (4)$$

The eqn $y = a \sin \frac{2\pi}{\lambda}(vt - x)$,
$y = a\sin(\omega t - kx)$ and $y = ae^{i(\omega t - kx)}$
denote a wave moving to the right along +X axis.
If the wave moves to the left (along −X direction), the sign ϕ changes because the oscillations at X begin before that at X=0 and the equations are represented as:

$$\left.\begin{array}{l} y = a \sin \frac{2\pi}{\lambda}(vt + x), \\ y = a\sin(\omega t + kx) \end{array}\right\} \longrightarrow (5)$$

and $$y = ae^{i(\omega t + kx)} \longrightarrow (6)$$

<u>Differential equation of wave Motion</u>

The displacement of a particle in a medium is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \longrightarrow (1)$$

where v is the velocity of the progressive wave

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wavelength and on moving from A to B, The phase changes by 2π. Therefore, on moving from point A to point P at a distance x from A, the phase changes and is given by ϕ-

$$\phi = \frac{2\pi}{\lambda} x$$

Hence the displacement of P is given by:

$$y = a \sin(\omega t - \phi)$$

$$= a \sin\left(2\pi n t - \frac{2\pi}{\lambda} x\right) = a \sin\left(2\pi \frac{v}{\lambda} t - \frac{2\pi}{\lambda} x\right)$$

or $$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{—— (1)}$$

The equation is quite general and gives the displacement of particles whose distance x from a fixed point A is known at any time. So ~~that~~ This is the equation for an increasingly simple harmonic wave. The number of wavelengths in a unit distance is called wave-number. It is denoted by $\bar{v}$.

$$\therefore \bar{v} = \frac{1}{\lambda} \quad \text{—— (2)}$$

The quantity $2\pi/\lambda = K$ is called Propagation constant.

Other form: equ^n (1) can also be written as

$$y = a \sin\left(\frac{2\pi v}{\lambda} t - \frac{2\pi}{\lambda} x\right) = a \sin(\omega t - Kx) \quad \text{—— (3)}$$

$$\left[\because \frac{2\pi v}{\lambda} = 2\pi n = \omega\right]$$