# Homogeneous and Non-Homogeneous Partial Differential Equations of Second and Higher Order with Constant Coefficients

**1. Introduction**
Partial Differential Equations (PDEs) of second and higher order with constant coefficients play an important role in mathematics, physics, and engineering. Such equations arise naturally in problems of heat conduction, wave propagation, elasticity, and potential theory. This document discusses homogeneous and non-homogeneous PDEs with constant coefficients and PDEs reducible to equations with constant coefficients, along with detailed methods of solution and illustrative examples.

**2. Linear PDE with Constant Coefficients**
A linear partial differential equation of order n with constant coefficients is of the form:
*F(D, D') z = ϕ(x, y)*
where $D = \partial/\partial x$ and $D' = \partial/\partial y$, F is a polynomial in D and D' with constant coefficients, and ϕ(x, y) is a given function. If ϕ(x, y) = 0, the equation is called homogeneous; otherwise, it is called non-homogeneous.

**3. Homogeneous Linear PDE with Constant Coefficients**
The general solution of a homogeneous linear PDE is obtained by finding the complementary function (C.F.). Assume a solution of the form z = e^(mx + ny). Substituting into the equation gives the auxiliary equation:
*F(m, n) = 0*. The nature of the roots of this equation determines the complementary function.

**Example 1:** Solve $(D^2 – 4DD' + 3D'^2)z = 0$.
Auxiliary equation: $m^2 – 4mn + 3n^2 = 0 = (m – n)(m – 3n)$.
Thus m = n and m = 3n.
Hence the complementary function is:
*z = ϕ■(y + x) + ϕ■(y + 3x)*, where ϕ■ and ϕ■ are arbitrary functions.

**4. Non-Homogeneous Linear PDE with Constant Coefficients**
The general solution of a non-homogeneous linear PDE consists of two parts:
(i) Complementary Function (C.F.), obtained from the associated homogeneous equation.
(ii) Particular Integral (P.I.), which depends on the form of the non-homogeneous term ϕ(x, y).
Thus, the complete solution is z = C.F. + P.I.

**Example 2:** Solve $(D^2 – D'^2)z = \sin(x + y)$.
Homogeneous equation: $(D^2 – D'^2)z = 0$.
Auxiliary equation: $m^2 – n^2 = 0 \Rightarrow m = \pm n$.
C.F.: z = ϕ■(x + y) + ϕ■(x – y).
For the P.I., since sin(x + y) is a function of (x + y), substitute $D = \partial/\partial x$ and $D' = \partial/\partial y$ to evaluate the operator. The particular integral is:
*P.I. = –(1/4) sin(x + y)*.
Hence the complete solution is:
*z = ϕ■(x + y) + ϕ■(x – y) – (1/4) sin(x + y)*.

**5. PDE Reducible to Equations with Constant Coefficients**
Some PDEs do not initially appear to have constant coefficients but can be transformed into equations with constant coefficients by suitable change of variables. Such transformations simplify the equation and allow the application of standard methods.

**Example 3:** Solve $x^2\, \partial^2 z/\partial x^2 - y^2\, \partial^2 z/\partial y^2 = 0$.
Let $x = e^u$ and $y = e^v$. Then the given equation reduces to a PDE with constant coefficients in u and v.
After transformation, the equation becomes:
$(D_u^2 - D_v^2)z = 0$.
Auxiliary equation: $m^2 - n^2 = 0 \Rightarrow m = \pm n$.
Hence the solution is:
$z = \phi$■$(u + v) + \phi$■$(u - v)$.
Rewriting in terms of x and y, we obtain the final solution.

## 6. Conclusion

Homogeneous and non-homogeneous linear PDEs with constant coefficients form the backbone of the theory of partial differential equations. The method of auxiliary equations provides a systematic approach to finding complementary functions, while particular integrals account for external forcing terms. PDEs reducible to constant coefficient form further extend the applicability of these methods, making them powerful tools in both theoretical and applied mathematics.