B.Sc Sem – IV, MJC-07

Monge's Method :— Monge's method of integrating $Rr+Ss+Tt=V$ where r, s, t are in the first degree and the Coefficients R, S, T, V are functions of p, q, x, y, z

The given equation is $Rr+Ss+Tt=V$ — (1)

where r, s, t have their usual meanings

We know that $dp = \frac{\partial p}{\partial x}\cdot dx + \frac{\partial p}{\partial y}\cdot dy = r\cdot dx + s\cdot dy$

and $dq = \frac{\partial q}{\partial x}\cdot dx + \frac{\partial q}{\partial y}\cdot dy = s\cdot dx + t\cdot dy$

$\therefore\ r = \dfrac{dp - s\,dy}{dx}$ and $t = \dfrac{dq - s\cdot dx}{dy}$

Substituting these values in (1)

We get

$$R\left(\frac{dp - s\,dy}{dx}\right) + Ss + T\left(\frac{dq - s\,dx}{dy}\right) = V$$

$$\Rightarrow R\cdot dy\,(dp - s\,dy) + Ss\,dx\,dy + T\,dx\,(dq - s\,dx) = V\,dx\,dy$$

$$\Rightarrow (R\,dp\,dy + T\,dq\,dx - V\,dx\,dy) = s\{R(dy)^2 - S\,dx\,dy + T(dx)^2\} = 0 \qquad (2)$$

The chief feature of Monge's method is obtaining one or two relations between p, q, x, y, z (Each relation involving an arbitrary function) to satisfy the simultaneous equations.

$$R\,dy^2 - S\,dy\,dx + T\,dx^2 = 0 \qquad (3)$$

$$R\,dp\,dy + T\,dq\,dx - V\,dy\,dx = 0 \qquad (4)$$

The equations (3) and (4) are called Monge's subsidiary equations.

Working method: step 1.

Let ③ being a quadratic be resolved into two linear equations in dx and dy in the form

$$dy - m_1 dx = 0 \quad \text{—— (5)}$$

and

$$dy - m_2 dx = 0 \quad \text{—— (6)}$$

Step 2. Now from ④ and ⑤ combined if necessary obtain two integrals.

$$u_1 = a \quad \text{and} \quad v_1 = b$$

then the relation $u_1 = F_1(v_1)$ —— (7)

is the solution and is called an intermediate integral

Similarly from ④ and ⑥ obtain another intermediate integral in the form $u_2 = F_2(u_2)$

Step 3. From ⑦ and ⑧ find the values of p and q in terms of x and y.

Step 4. Substitute these values of p and q in $dz = p.dx + q.dy$ and integrate it. The complete integral of the equation ① is thus obtained.

Note:— Sometimes when the linear factors of the equation ③ is a perfect square it becomes easier to find the complete integral with the help of one intermediate integral only. This is done with the help of Lagrange's method. The method of procedure will be best understood by studying worked examples.