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4) Phase change with Time:

In case a particle at a distance x from origin has phase ϕ_1 at t_1 and ϕ_2 at t_2 Then we have

$$\phi_1 = (\omega t_1 - kx) \text{ and } \phi_2 = (\omega t_2 - kx)$$

$$\therefore \phi_1 - \phi_2 = \omega(t_1 - t_2)$$

Thus, the phase difference is given by

$$\Delta\phi = \frac{2\pi}{T} \times \text{Time difference } (\Delta t)$$

5) The velocity of a particle at a given time is equivalent to the product of wave velocity and -ve of slope of the wave curve at the given position and time.

$$v_{\text{particle}} = -v \left(\partial y / \partial x \right)$$

6) Acceleration of a particle at (x, t) is $a = \frac{d^2 y}{dt^2}$
also, $|a_{\text{max}}| = \omega^2 A$

7) Intensity:

Intensity is defined as an energy flow of a wave per unit area of cross-section of the medium in unit time.

Intensity for the wave through a string is given as

$$I = \frac{\text{Power}}{\text{Area of cross-section}} = \frac{P}{S} = \frac{1}{2} \rho \omega^2 A^2 v$$

Where, $\rho = \text{density of string}$
 $v = \text{speed of the wave}$

— (1)



Intensity of Sound wave is specified as

$$I = \frac{P_0^2}{2\rho v} \quad \text{--- (2)}$$

Where, P_0 = Pressure amplitude
 ρ = density of the medium
 v = Speed

on Comparing eqnⁿ (1) and eqnⁿ (2), we get

$$P_0 = A\rho v\omega$$

Example. The human-ear can detect the faintest sound at a frequency of 1 KHz (corresponds to an intensity of about 10^{-12} W/m^2 (that so called threshold of hearing)). Find the pressure amplitude and maximum displacement associated with this sound given that the density of air = 1.3 Kg/m^3 and velocity of sound in air is 332 m/s .

Sol. Given that the density of air = 1.3 Kg/m^3
 velocity of sound in air is 332 m/s .

We know that $I = P_0^2 / 2\rho v$

$$\Rightarrow P_0 = \sqrt{I \times 2\rho v}$$

$$\text{or } \Rightarrow P_0 = \sqrt{(10^{-12}) \times 2 \times 1.3 \times 332} = 2.94 \times 10^{-5} \text{ N/m}^2$$

Now, as $P_0 = \rho v \omega A$

$$\begin{aligned} \Rightarrow A &= \frac{P_0}{\rho v \omega} = \frac{2.94 \times 10^{-5}}{1.3 \times 332 \times (2\pi \times 10^3)} \\ &= 1.1 \times 10^{-11} \text{ m} \end{aligned}$$

Energy Transport of a wave

When a wave is set up on a stretched string, it provides energy for the motion of the string. Wave transports both kinetic as well as elastic potential energy when the wave moves away from one end to other end.

Oscillating string element holds both maximum kinetic as well as maximum elastic potential energy at $y=0$. Fig-9 shows to the snapshot of the regions of the string at maximum displacement have no energy, and the regions at zero displacement have maximum energy, when the wave travels with the string then the forces in the string continuously applied due to the tension in order to transfer energy from regions with high energy to regions with low energy.

A wave is transmitted along the string by continuously oscillating one end of the string. By doing this we continuously impart energy to the string's motion and stretching of the string since the sections of string oscillate perpendicular to the x -axis, because they have kinetic energy and elastic potential energy. Therefore the energy is transferred into these new sections as the wave moves into sections that were previously

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at rest. Hence, the wave transfers the energy along the string.

Kinetic Energy:

Let us consider a string element whose mass is dm and it is oscillating transversely in simple harmonic motion just as wave passes through it and also, the kinetic energy that is associated with its transverse velocity u .

Fig-9 (element b) shows that when element is rushing through its $y=0$ position its transverse velocity and also its kinetic energy will be maximum.

Elastic Potential Energy:

In order to transmit a sinusoidal wave along a previous straight string should be stretch the string. Suppose that a string element of length ' dx ' oscillates transversely if the string element is to fit the sinusoidal wave form then its length its length must increase and decrease in a periodic manner. Elastic potential energy is associated with the length changes as for a spring. (27)



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Fig-9 Shows That if There is a string element at its position $y = y_m$ and dx be the undisturbed length Then its elastic Potential energy is Zero. Element can have the maximum stretch if The element is rushing Through its $y = 0$ position and hence maximum elastic Potential energy.

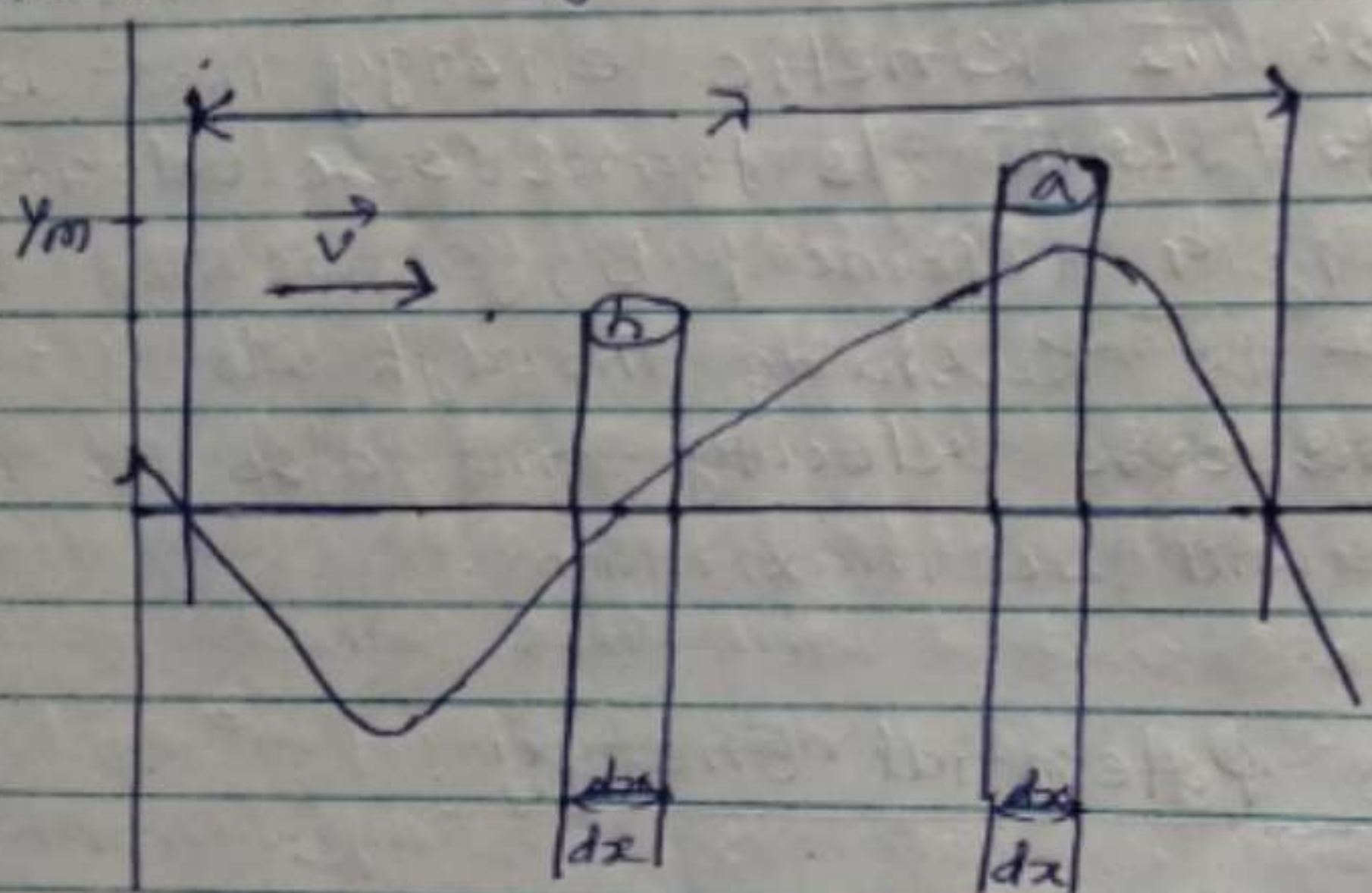


Fig-9 diagram of a Travelling wave on a String at time $t=0$. String element 'a' is at displacement $y = y_m$, and String element 'b' is at displacement $y = 0$. The Kinetic energy of the String element at each position depends on the Transverse velocity of the element. The Potential energy depends on the Amount by which the String element is stretched as the wave passes through it.