VKSU

Group velocity

Group velocity is defined as the velocity in which the wave packet travels in a dispersive medium. Usually the wave group has the maximum amplitude at a particular value of 'x' and the velocity of this maximum amplitude point is known as the group velocity.

Therefore, the velocity at which a wave group (or a pulse) travels is the group velocity of the wave group. This velocity also signifies the velocity in which energy of the wave group is transmitted.

Expression for group velocity!

Let us consider a wave packet which has a group of two waves slightly different in angular frequencies and phase velocities but of equal amplitude for determining the expression for group velocity of a wave packet.

When their angular frequency are ω_1 and ω_2 propagation constants are k_1 and k_2 and the amplitude is a then their separate displacement at any instant t can be represented as:

$$Y_1 = a \sin(\omega_1 t - K_1 x) \quad \text{—} (1)$$

and $$Y_2 = a \sin(\omega_2 t - K_2 x) \quad \text{—} (2)$$

From Young's Principle of superposition, the resultant displacement Y at any instant t specified as

$$Y = Y_1 + Y_2 = a \sin(\omega t - K_1 x) = a \sin(\omega_2 t - K_2 x)$$

By using trigonometric relation, we get

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$, we get

$$Y = 2a \sin\left[\frac{(\omega_1+\omega_2)t}{2} - \frac{(K_1+K_2)x}{2}\right] \cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(K_1-K_2)x}{2}\right]$$

or

$$Y = 2a \cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(K_1-K_2)x}{2}\right] \sin\left[\frac{(\omega_1+\omega_2)t}{2} - \frac{(K_1+K_2)x}{2}\right] \quad \text{—} (3)$$

or $$Y = A \sin\left[\frac{(\omega_1+\omega_2)t}{2} - \frac{(K_1+K_2)x}{2}\right] \quad \text{—} (4)$$

Here $$A = 2a \cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(K_1-K_2)x}{2}\right]$$

Now, A is the modified amplitude of the wave packet. This modified amplitude of the wave packet is modulated in space and time both with a help of a very slowly varying envelope of frequency $(\omega_1 - \omega_2)/2$ and Propagation constant $(K_1 - K_2)/2$.

Its maximum value in $2a$.

VKSU equ^n (3) can also be written as :

$$y = 2a\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta K}{2}x\right]\sin(\omega t - Kx) \quad \text{--- (5)}$$

where,

$\omega_1 - \omega_2 = \Delta\omega$, $K_1 - K_2 = \Delta K$,

$\omega = \frac{\omega_1 + \omega_2}{2}$ and $K = \frac{K_1 + K_2}{2}$.

The group velocity V_g is the velocity in which this wave packet is represented by,

$$V_g = \frac{\omega_1 - \omega_2}{K_1 - K_2} = \frac{\Delta\omega}{\Delta K} \quad \text{--- (6)}$$

when a bunch of waves has a Number of frequency components in an infinitely small frequency interval Then equ^n (6) can be again written as :

$$V_g = \frac{d\omega}{dK} \quad \text{--- (7)}$$

equ^n (7) is equation of group velocity. This group velocity is also known as Particle velocity.

Ex. when the frequency of matter wave is $\frac{1}{h}\left(\frac{1}{2}mv^2\right)$ and the wavelength is $\frac{h}{mv}$, find the group velocity.

Sol: Given, $n = \frac{1}{h}\left(\frac{1}{2}mv^2\right)$ and $\lambda = \frac{h}{mv}$

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we know That, group velocity

$$(V_g) = \frac{\omega}{k} = \frac{2\pi n \lambda}{2\pi} = n\lambda \quad \text{--- (1)}$$

Putting The value of 'n' and λ in equ^n (1) we get

$$V_g = \frac{1}{h}\left[\frac{1}{2}mv^2\right] \times \frac{h}{mv} = \frac{v}{2}$$