Cyclic groups

Definition:— A group is said to be cyclic if it is capable of being generated by a single element.

This single element is called the generater of the group.

If a cyclic group is generated by an element a, then we can write $G = \{a\}$.

It is not necessary that all the elements of a cyclic group are distinct.

Alternate definition of cyclic groups:

A group G is said to be cyclic group if every element $x \in G$ is expressible as $x = a^n$ for some $n \in Z$.

a being an element of G.

The elements of G will be of the form.

$$\ldots, a^{-3}, a^{-2}, a^{-1}, a^0 = e, a, a^2, a^3, \ldots$$

A group $(G, +)$ is said to be cyclic if every $x \in G$ is expressible as $x = na$ for some integer n.

The elements of $(G, +)$ will be of the form

$$\ldots, -3a, -2a, -a, 0 = e, a, 2a, 3a, \ldots$$

Examples:— (i) The group $(Z, +)$ is cyclic and its generator is 1. Another generator is -1.

(ii) The multiplicative group $\{1, \omega, \omega^2\}$ is cyclic and generators are ω and ω^2.

(iii) The multiplicative group of 5th roots of unity is a cyclic group and its generator is $e^{2\pi i/5}$.

(iv) The multiplicative group of n, nth roots of unity is cyclic and its generator is $e^{2\pi i/n}$.

(v) Let $G = \{0, 1, 2, 3, 4, 5\}$. Then $(G, +_6)$ is a cyclic group and its generator is 1.

(vi) Let $G = \{1, 2, 3, 4, 5, 6\}$. Then $(G, \times_7)$ is cyclic and its generators is 3.