

Limit of Functions

In calculus, the concept of a limit is fundamental. The limit of a function describes the behavior of the function as the input variable approaches a particular value. Even if the function is not defined at that value, the limit may still exist if the function values approach a specific number.

Definition of Limit

Let $f(x)$ be a function defined in the neighborhood of a point a (except possibly at a itself). We say that the limit of $f(x)$ as x approaches a is L , written as $\lim_{x \rightarrow a} f(x) = L$, if the value of $f(x)$ can be made arbitrarily close to L by taking x sufficiently close to a .

Example (Detailed)

Find the limit of the function $f(x) = (x^2 - 1)/(x - 1)$ as x approaches 1.

Step 1: Direct substitution

Substitute $x = 1$ into the function:

$(1^2 - 1)/(1 - 1) = 0/0$, which is indeterminate.

Step 2: Simplify the function

Factor the numerator:

$$x^2 - 1 = (x - 1)(x + 1)$$

So, $f(x) = (x - 1)(x + 1)/(x - 1)$.

Cancel the common factor $(x - 1)$:

$f(x) = x + 1$, for $x \neq 1$.

Step 3: Evaluate the limit

Now substitute $x = 1$ into the simplified expression:

Limit = $1 + 1 = 2$.

Hence, $\lim_{x \rightarrow 1} (x^2 - 1)/(x - 1) = 2$. Although the function is not defined at $x = 1$, the limit exists and is equal to 2.

Conclusion

Limits help us understand the behavior of functions near specific points and form the foundation of continuity, differentiation, and integration in calculus.