Dr. Anupama Sinha

Assistant Professor
HD Jain College, Mathematics Department, Ara

FOURIER SERIES (Interval C to C + 2π)

1. Introduction:

A Fourier series represents a periodic function as a sum of sine and cosine terms.

If f(x) is defined in the interval (C, C + 2π), then its Fourier series is:

f(x) = a0/2 + Σ [an cos n(x-C) + bn sin n(x-C)], n=1 to ∞

Where,

a0 = (1/π) ∫ from C to C+2π f(x) dx

an = (1/π) ∫ from C to C+2π f(x) cos n(x-C) dx

bn = (1/π) ∫ from C to C+2π f(x) sin n(x-C) dx

----

2. Dirichlet's Conditions:

A function f(x) can be expanded in Fourier series if:

1. f(x) is periodic.
2. f(x) is single valued and finite.
3. f(x) has finite number of discontinuities.
4. f(x) has finite number of maxima and minima.
5. f(x) is absolutely integrable over one period.

If these conditions are satisfied, Fourier series converges to:

- f(x) at points of continuity.
- (f(x+) + f(x-))/2 at points of discontinuity.

----

3. Euler's Formula:

Euler's relations:

e^(ix) = cos x + i sin x

e^(-ix) = cos x - i sin x

Thus,

$\cos x = (e^{ix} + e^{-ix})/2$

$\sin x = (e^{ix} - e^{-ix})/(2i)$

Complex form of Fourier series:

$f(x) = \Sigma C_n e^{inx}$

Where,

$C_n = (1/2\pi) \int f(x) e^{-inx} dx$

----

4. Fourier Series for Even and Odd Functions:

If f(x) is Even:

$b_n = 0$

Fourier series contains only cosine terms.

If f(x) is Odd:

$a_n = 0$

Fourier series contains only sine terms.

----

5. Examples (General Interval):

Example 1:

Find Fourier series of $f(x)=x$ in $(0,2\pi)$.

Example 2:

Find Fourier series of $f(x)=x^2$ in $(-\pi,\pi)$.

Example 3:

Expand $f(x)=|x|$ in $(-\pi,\pi)$.

Example 4:

Expand $f(x)=\sin x$ in $(-\pi,\pi)$.

Example 5:

Expand $f(x)=\cos x$ in $(-\pi,\pi)$.

----

6. Even Function Examples:

1. $f(x)=x^2$
2. $f(x)=|x|$
3. $f(x)=\cos x$

4. $f(x)=\cos 2x$

5. $f(x)=e^x + e^{-x}$

----

7. Odd Function Examples:

1. $f(x)=x$
2. $f(x)=\sin x$
3. $f(x)=\sin 2x$
4. $f(x)=x^3$
5. $f(x)=x \sin x$

Conclusion:

Fourier series is a powerful tool for representing periodic functions in mathematical physics and engineering.