Resultant Intensity Interference of two waves

According to the given fig, suppose y_1 and y_2 two waves of vertical displacements that superimpose at a point P in space, the resultant displacement is represented as

$$Y = Y_1 + Y_2$$

At the same time, these waves meet at some point P, and the only difference occurs in their phases.

$$Y_1 = a \sin$$

$$Y_2 = b \sin(\omega t + \phi)$$

where, a and b are their respective amplitudes. ϕ is the constant phase difference between the two waves.

On applying superposition principle, we get

$$Y = a \sin \omega t + b \sin(\omega t + \theta) \quad \text{—— (1)}$$

Equation (1) is represented as the phasor diagram the resultant having amplitude A and a phase angle with respect to y_1 is given as

$$Y = A \sin(\omega t + \theta)$$

$$A \sin(\omega t + \theta) = a \sin \omega t + b \sin(\omega t + \theta)$$

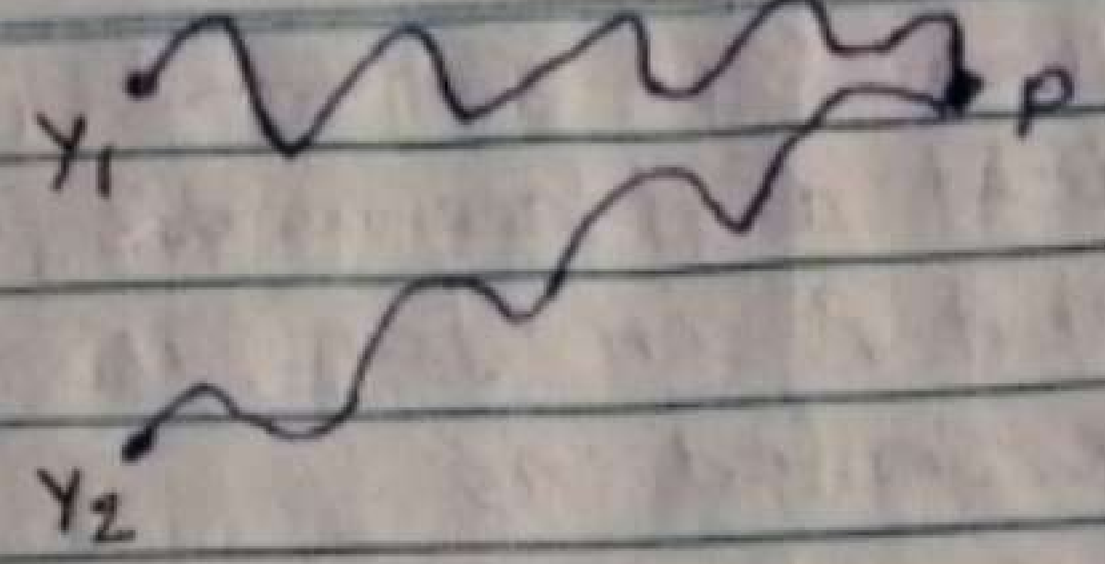
using $[\sin(A+B) = \sin A \cos B + \cos A \sin B]$

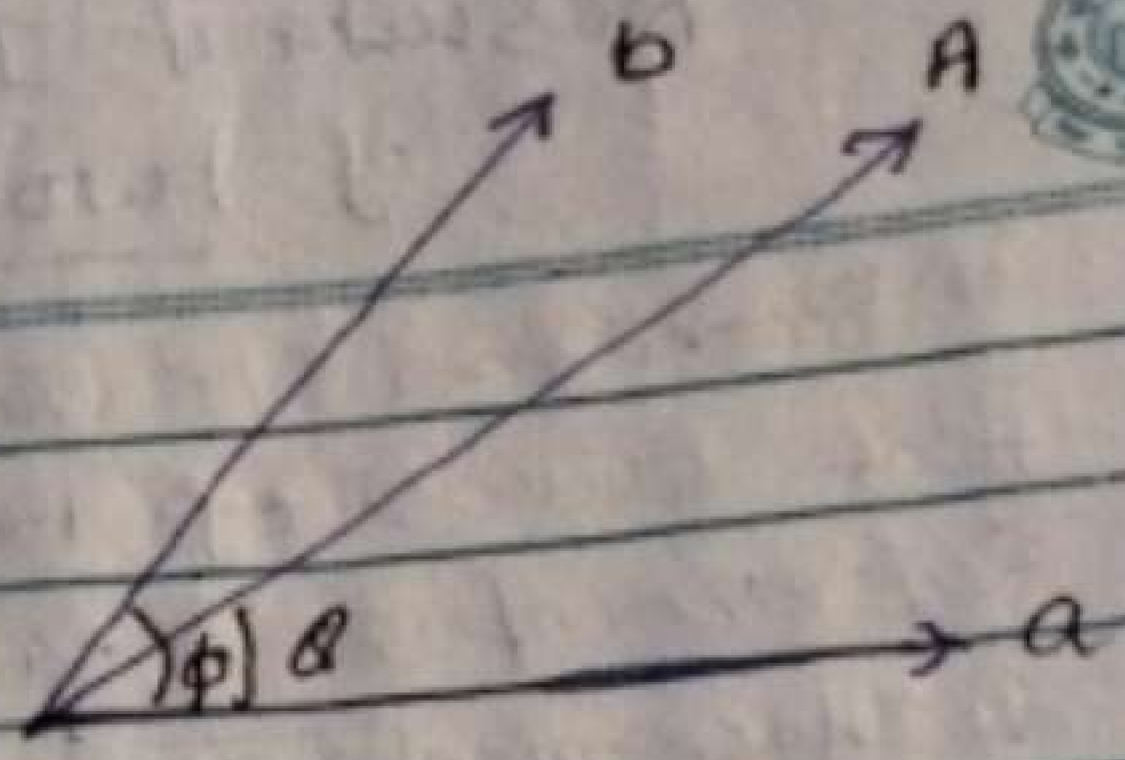
$$A[\sin Wt \cos \theta + \cos Wt \sin \theta] = a \sin Wt + b[\sin Wt \cos \theta + \cos Wt \sin \theta]$$

By equating '$\sin \omega t$' and '$\cos \omega t$' terms on both sides, we get

$$A \cos \theta = a + b \cos \theta \quad \text{—— (2)}$$

$$A \sin \theta = b \sin \theta \quad \text{—— (3)}$$





on Squaring and adding equn (2) and equn (3), we get

$$A^2 = (a + b\cos\theta)^2 + (b\sin\theta)^2$$

$$A^2 = a^2 + b^2 + 2ab\cos\theta$$

$$A = \sqrt{a^2 + b^2 + 2ab\cos\theta} \quad \text{—— (4)}$$

If we divide equn (2) and equn (3), we get

$$\tan\theta = \frac{b\sin\theta}{a + b\cos\theta} \quad \text{—— (5)}$$

Since intensity is indirectly proportional to the square of the amplitude of the wave

$$I \propto a^2$$

$$I_1 = ka^2, \quad I_2 = kb^2, \quad I = kA^2$$

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Taking

$$I_1 = KA^2$$

$$I = K(a^2 + b^2 + 2ab\cos\theta)$$

$$I = Ka^2 + Kb^2 + 2Kab\cos\theta$$

$$I = Ka^2 + Kb^2 + Kab\cos\theta$$

$$I = I_1 + I_2 + 2(\sqrt{K}a)(\sqrt{K}b)\cos\theta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta \quad \text{—— (6)}$$

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when $\cos\theta = \pm 1$ then intensity should be maximum ($I = I_{max}$) for constructive interference.

$\theta = 0, 2\pi, 4\pi, \ldots, (2n\pi)$

If the path difference between the waves at point P is Δx, we have

$\Delta x = \frac{\lambda}{2\pi}(\theta)$; $\Delta x = \frac{\lambda}{2\pi}(2n\pi)$

$n = 1, 2, 3, \ldots$

$\Delta x = n\lambda$

<u>Condition for Constructive Interference:</u>

Phase difference $= 0$ or $2n\pi$ where $n = 1, 2, 3, \ldots$

Path difference $= n\lambda$ where $n = 1, 2, 3, \ldots$

During constructive interference $I = I_{max}$, Then

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

$= I_{max} = K(a+b)^2$

when $\cos\phi = -1$, intensity should be minimum ($I = I_{min}$) for destructive interference.

$\phi = \pi, 3\pi, 5\pi, \ldots$

$\Rightarrow \phi = (2n-1)\pi$, when $n = 1, 2, 3, \ldots$

of Δx is the path difference between the waves at point P.

$\Delta x = \frac{\lambda}{2\pi}\phi$; $\Delta x = \frac{\lambda}{2\pi}(2n-1)\pi$

Thus, we have $\Delta x = \frac{(2n-1)}{2}\lambda$

Condition for Destructive Interference

Phase difference $= (2n-1)\pi$

Path difference $= (2n-1)\lambda/2$

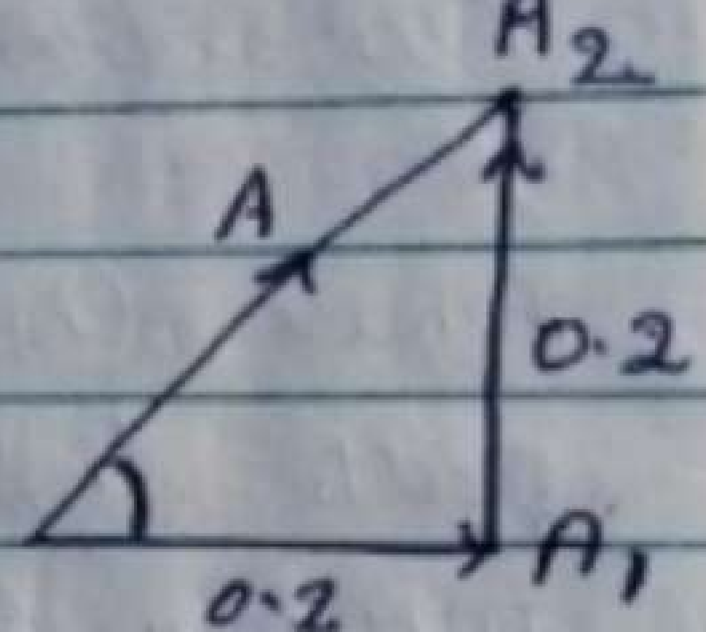
$I = I_{min}$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$

Ex The two harmonic waves $y_1(x,t) = 0.2\sin(x-3t)$ and $y_2(x,t) = 0.2\sin(x-3t+\phi)$ are given. If $\phi = \pi/2$ rad then determine the expression of y. If ϕ between the waves is unknown and the amplitude of their sum is 0.32 m, then determine the value of ϕ.

Sol: Given that $y_1(x,t) = 0.2\sin(x-3t)$

$y_2(x,t) = 0.2\sin(x-3t+\phi)$



Since $y = y_1 + y_2$ then we have

$= 0.2\sin(x-3t) + 0.2\sin(x-3t+\phi)$

$= 0.2\sin(x-3t) + 0.2\sin\left(x-3t+\frac{\pi}{2}\right)$

$= A\sin(x-3t+\theta)$

$A = \sqrt{(0.2)^2 + (0.2)^2} = 0.283\,m$

$\theta = \tan^{-1}\frac{0.2}{0.2} = \pi/4$

Thus the expression of y is

$\therefore \cancel{y = 0.28\sin}\left(x - 3t + \frac{\pi}{4}\right)$

If amplitude of resulting wave is 0.32 m and A = 0.2 m, then

$0.32 = \sqrt{(0.2)^2 + (0.2)^2 + 2(0.2)(0.2)\cos\phi}$

$\Rightarrow \phi \pm 1.29$ rad