Hence

$$\alpha = \frac{e^{-V_i P/L E}}{L/p} = \frac{1}{L} e^{-V_i P/L E}$$

Putting $A = 1/L_0$ and $B = \frac{V_i}{L_0}$; we get

$$\frac{\alpha}{P} = A \exp\left[\frac{-B}{(E/P)}\right]$$

where A and B are the two constants introduced by Townsend and which vary from gas to gas. Townsend next argued that had this been the only process the variation of the current with the voltage would have been represented by the equation $i = i_0 e^{\alpha d}$ but actually the current rises very rapidly beyond a certain applied voltage. Townsend made the assumption that at this stage secondary electrons are generated by the positive ions themselves as they impinge on the cathode surface. Besides the positive ions, photons incident on the cathode surface can also generate secondary electrons.

Since $n = n_0 e^{\alpha d}$, the number of electrons produced by a single electron is $e^{\alpha d}$ and the number of positive ions produced is $(e^{\alpha d} - 1)$. These $(e^{\alpha d} - 1)$ positive ions move towards the cathode and impinge on it. If γ is the number of electrons that are released from the cathode due to the incidence of a single positive ion the number of electrons generated is $\gamma(e^{\alpha d} - 1)$. These electrons will now produce $\gamma(e^{\alpha d} - 1)$ electrons due to collision ionization in space. So the total number of electrons now present in space is

$$\gamma(e^{\alpha d} - 1) + \gamma(e^{\alpha d} - 1)^2$$

$$= \gamma[e^{\alpha d} - 1] + \gamma[e^{2\alpha d} - 2e^{\alpha d} + 1]$$

$$= \gamma[e^{\alpha d} - 1 + e^{2\alpha d} - 2e^{\alpha d} + 1]$$

$$= \gamma[e^{2\alpha d} - e^{\alpha d}]$$

$$= \gamma e^{\alpha d}[e^{\alpha d} - 1]$$