Equation of Beats

Suppose There are two wave trains having frequency n_1 and n_2 where $(n_1 - n_2)$ is small. Assume That a and b are The amplitudes of The waves respectively. Simply we can assume That The two waves are in Phase at any Point in The medium at $t = 0$. The displacement Y_1 and Y_2 due to each wave are specified as

$$Y_1 = a \sin \omega_1 t$$
$$Y_2 = b \sin \omega_2 t$$

Here $\omega_1 = 2\pi n_1$

and $\omega_2 = 2\pi n_2$

$$\therefore \quad Y_1 = a \sin 2\pi n_1 t \qquad \text{— (1)}$$

$$\text{and} \quad Y_2 = b \sin 2\pi n_2 t \qquad \text{— (2)}$$

The resultant displacement is specified as

$$Y = Y_1 + Y_2$$
$$Y = a \sin 2\pi n_1 t + b \sin 2\pi n_2 t$$
$$Y = a \sin 2\pi n_1 t + b \sin 2\pi [n_1 - (n_1 - n_2)] t$$
$$Y = a \sin 2\pi n_1 t + b [\sin 2\pi n_1 t \cos 2\pi (n_1 - n_2) t - \cos 2\pi n_1 t \sin 2\pi (n_1 - n_2) t] \qquad \text{— (3)}$$

$$Y = \sin 2\pi n_1 t \, [a + b \cos 2\pi (n_1 - n_2) t] - \cos(2\pi n_1 t) \, [b \sin 2\pi (n_1 - n_2) t]$$

Take $a + b \cos 2\pi (n_1 - n_2) t = A \cos\theta$

And $b \sin 2\pi (n_1 - n_2) t = A \sin\theta$

$$\therefore \quad Y = A \sin(2\pi n_1 t - \theta)$$

Here $\tan\theta = \dfrac{b\sin 2\pi(n_1-n_2)t}{a+b\cos 2\pi(n_1-n_2)t}$ —— (4)

and

$A = \sqrt{a^2+b^2+2ab\cos 2\pi(n_1-n_2)t}$ —— (5)

Equation (4) specifies that the phase angle θ changes with respect to time. Similarly eqn (5) specifies that the amplitude of the resultant vibration also changes with time.

1.) When $2\pi(n_1-n_2)t = 2k\pi$
where $K = 0, 1, 2, 3, \ldots$ etc.
The resultant amplitude
$A = \sqrt{a^2+b^2+2ab}$
$A = (a+b)$ ——— (6)
If $t = \dfrac{K}{(n_1-n_2)}$, The resultant amplitude is defined as maximum.

~~If $t = \frac{K}{(n_1}$~~

i.e. at time instant, $0, \dfrac{1}{(n_1-n_2)}, \dfrac{2}{(n_1-n_2)}$ etc. the amplitude of the resultant is maximum. The maximum sound intensity will be audible during the such instants since sound intensity is directly proportional

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to the square of amplitude.

$0, \frac{1}{(n_1-n_2)}, \frac{2}{(n_1-n_2)}, \frac{3}{(n_1-n_2)} \ldots$ etc.

2.) When $2\pi(n_1-n_2)t = (2K+1)\pi$

where $K = 0, 1, 2, 3$, etc.

The resultant amplitude $A = \sqrt{a^2+b^2-2ab}$

$A = (a-b)$

If $t = \frac{(2K+1)}{2(n_1-n_2)}$, the resultant amplitude is defined as maximum that means at time instants.

$\frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)} \ldots$ etc.

the amplitude of the resultant is minimum. Since the Minimum intensity of second will be heard at the instant, so

$\frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)}$ etc.

Hence, the maxima and minima occur alternately after equal intervals of time $\frac{1}{2(n_1-n_2)}$.

The time interval between two successive maxima or between two successive minima is $\frac{1}{(n_1 - n_2)}$.

We know that the number of beats produced per second is $(n_1 - n_2)$. In case the amplitude of the two wave trains are equal, the maximum resultant amplitude is $2a$ and the minimum amplitude is 0. Here, the intensity of minima positions will be zero.